'Hyperbolicity of complex varieties'

Winter term 2016/17, Tuesday 2-4pm, 0.011

The aim of the seminar is to understand the ideas behind recent works on Kobayashi hyperbolicity and Green-Griffiths-Lang conjecture. We will start from the classical material on the relations between different notions of hyperbolicity for complex varieties, discuss basic examples and constructions. Then we will study more advanced techniques developed by Green-Griffiths, Demailly, McQuillan, Siu and others, discuss applications of these techniques to hyperbolicity of surfaces.

For further information or if you want to give a talk in the seminar, please contact huybrech@ or aosoldat@. The **first talk will be on October 25.**

25 October: Introduction (Speaker: Andrey Soldatenkov)
A general introduction to the topic of hyperbolicity of complex varieties will be given, following mostly the survey paper of Lang [25] and introductory chapters of Demailly's [13]. We will discuss various notions of hyperbolicity and relations between them, prove some basic properties. The talk is expected to be elementary.

15 November: Varieties with ample cotangent bundle (Speaker: Emma Brakkee)
The talk should follow the exposition in Debarre's notes [7]. After presenting some elementary preliminary material (the notion of ampleness for vector bundles, Ahlfors-Schwarz lemma), it should be proven that varieties with ample cotangent bundle are hyperbolic. After that various constructions of varieties with ample cotangent bundle should be discussed, following the same notes by Debarre and his paper [8].

22 November: Introduction to jet differentials (Speaker: Francesco Genovese)
Jet differentials are the technical heart of modern approaches to hyperbolicity, so we should spend some time studying them. The idea of using jets goes back to A. Bloch. We should start to discuss Part A of the Green-Griffiths paper [23] (skipping the piece about surfaces of general type). The following things should be explained: jet bundles, jet differentials, k-jet metrics with negative curvature. One can follow the more detailed exposition in [11], §§6–8. Some parts can be postponed until the next talk.

29 November: Jet differentials and Bloch's theorem (Speaker: Daniel Huybrechts)
This should be a continuation of the previous talk. After finishing the introduction to jet differentials, the proof of A. Bloch's theorem should be explained. One can follow either [23] or [11], §10. Other references include [18] and [27].

13 December: Curves on surfaces of general type (Speaker: Vladimir Lazić)
Bogomolov has proven in [2] that on a surface of general type with $c_1^2 > c_2$ curves of any fixed geometric genus form a bounded family. An overview of this and related results is given in [3] and [9]. We can also follow section 7 in [7]. The proof uses some

foliation theory. Then we can discuss the theorem of Lu-Yau about entire curves on surfaces of general type with $c_1^2 > 2c_2$ which can also be found in [7].

10 January: Algebraic hyperbolicity of hypersurfaces in projective space (Speaker: Corinne Bedussa)
Clemens has shown in [6] that a general hypersurface in $\mathbb{P}^n$ of degree $\geqslant 2n-1$ does not contain rational curves. His results have been subsequently improved by Ein [20], [21], Voisin [34] and Pacienza [30]. In fact, it follows from these works that a general hypersurface in $\mathbb{P}^n$ of sufficiently high degree is algebraically hyperbolic. The aim of the talk is to explain this result. One can follow one of the mentioned papers or section 6 of [7], the notes [35] can also be useful. The proofs here are purely algebraic and do not use any analytic techniques.

17 January: Hyperbolic surfaces in $\mathbb{P}^3$ I (Speaker: ?)
24 January: Hyperbolic surfaces in $\mathbb{P}^3$ II (Speaker: ?)
It is conjectured that a generic surface in $\mathbb{P}^3$ of degree $\geqslant 5$ is hyperbolic. Weaker bounds have been obtained by Demailly-El Goul [10] (degree $\geqslant 21$) and Paun [31] (degree $\geqslant 18$). The talks should give a survey of the paper [10], with the ideas of the proofs if possible. An independent approach has been developed by McQuillan [28]. Other references are [31] and [18]. Another possible topic is the construction of explicit examples of hyperbolic surfaces of low degree: see [19] (degree 6) or section 5 in [7] (degree 8). See also [29], [22].

31 January: Complex surfaces dominated by $\mathbb{C}^2$ (Speaker: Stefan Schreieder)
The talk should give an overview of results of Buzzard and Lu [4]. It would be especially interesting to understand examples of K3 surfaces dominated by $\mathbb{C}^2$ (section 4 in [4]). The paper [32] may be helpful.

7 February: (The topic is open for discussion) (Speaker: ?)

References

[1] F. Bogomolov, B. De Oliveira *Hyperbolicity of nodal hypersurfaces.* J. Reine Angew. Math. 596 (2006), 89–101.

[2] F. Bogomolov *Families of curves on a surface of general type.* Soviet Math. Dokl. 18 (1977), 1294–1297.

[3] M. Brunella *Courbes entières dans les surfaces algébriques complexes (d'après McQuillan, Demailly-El Goul,).* Séminaire Bourbaki, Vol. 2000/2001. Astérisque No. 282 (2002), Exp. No. 881, vii, 39–61.

[4] G. Buzzard, St. Lu *Algebraic surfaces holomorphically dominable by* $\mathbb{C}^2$. Invent. Math. 139 (2000), no. 3, 617–659.

[5] S. Cantat *Deux exemples concernant une conjecture de Serge Lang.* C. R. Acad. Sci. Paris Sér. I Math. 330 (2000), no. 7, 581–586.

[6] H. Clemens *Curves on generic hypersurfaces.* Ann. Sci. Ec. Norm. Sup. 19 (1986), 629–636.

[7] O. Debarre *Hyperbolicity of complex varieties,*
`http://www.math.ens.fr/~debarre/DebarreCourse2.pdf`

[8] O. Debarre *Varieties with ample cotangent bundle* Comp. Math. 141 (2005), 1445–1459., Erratum: Compos. Math. 149 (2013), 505–506.

[9] M. Deschamps *Courbes de genre géométrique borné sur une surface de type général [d'aprés F. A. Bogomolov].* Séminaire Bourbaki, 30e anne (1977/78), Exp. No. 519, pp. 233–247

[10] J.-P. Demailly, J. El Goul *Hyperbolicity of generic surfaces of high degree in projective 3-space.* Amer. J. Math. 122 (2000), no. 3, 515–546.

[11] J.-P. Demailly *Hyperbolic algebraic varieties and holomorphic differential equations.* Acta Math. Vietnam. 37 (2012), no. 4, 441–512.

[12] J.-P. Demailly *Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials.* Algebraic geometry–Santa Cruz 1995, 285–360, Proc. Sympos. Pure Math., 62, Part 2, Amer. Math. Soc., Providence, RI, 1997.

[13] J.-P. Demailly *Recent progress towards the Kobayashi and Green-Griffiths-Lang conjectures,* `https://www-fourier.ujf-grenoble.fr/~demailly/manuscripts/takagi16_jpd.pdf`

[14] S. Diverio, St. Trapani *A remark on the codimension of the Green-Griffiths locus of generic projective hypersurfaces of high degree.* J. Reine Angew. Math. 649 (2010), 55-61.

[15] S. Diverio, J. Merker, E. Rousseau *Effective algebraic degeneracy.* Invent. Math. 180 (2010), no. 1, 161–223.

[16] S. Diverio *Existence of global invariant jet differentials on projective hypersurfaces of high degree.* Math. Ann. 344 (2009), no. 2, 293–315.

[17] S. Diverio *Differential equations on complex projective hypersurfaces of low dimension.* Compos. Math. 144 (2008), no. 4, 920–932.

[18] S. Diverio, E. Rousseau: *A survey on hyperbolicity of projective hypersurfaces.* IMPA Mathematical Publications, Rio de Janeiro, 2011. x+109 pp.

[19] J. Duval *Une sextique hyperbolique dans* $\mathbb{P}^3(\mathbb{C})$. Math. Ann. 330 (2004) 473–476.

[20] L. Ein *Subvarieties of generic complete intersections.* Invent. Math. 94 (1988), 163–169.

[21] L. Ein *Subvarieties of generic complete intersections, II.* Math. Ann. 289 (1991), 465–471.

[22] J. El Goul *Algebraic families of smooth hyperbolic surfaces of low degree in* $\mathbb{P}^3(\mathbb{C})$. Manuscripta Math. 90 (1996), 521–532.

[23] M. Green, Ph. Griffiths *Two applications of algebraic geometry to entire holomorphic mappings.* The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979), pp. 41–74, Springer, New York-Berlin, 1980.

[24] S. Kobayashi *Hyperbolic complex spaces,* Grundlehren der Mathematischen Wissenschaften, Vol. 318, Springer-Verlag, Berlin, 1998.

[25] S. Lang *Hyperbolic and Diophantine analysis.* Bull. Amer. Math. Soc. 14 (1986) 159–205.

[26] S.S.Y. Lu, S.-T. Yau *Holomorphic curves in surfaces of general type.* Proc. Nat. Acad. Sci. USA, 87 (January 1990), 80–82.

[27] M. McQuillan *A new proof of the Bloch conjecture.* J. Alg. Geom. 5 (1996), 107–117.

[28] M. McQuillan *Holomorphic curves on hyperplane sections of 3-folds.* Geom. Funct. Anal. 9 (1999) 370–392.

[29] A. Nadel *Holomorphic curves on hyperplane sections of 3-folds.* Geom. Funct. Anal. 9 (1999) 370392.

[30] G. Pacienza *Subvarieties of general type on a general projective hypersurface.* Trans. Amer. Math. Soc. 356 (2004), 2649–2661.

[31] M. Paun *Techniques de construction de différentielles holomorphes et hyperbolicité (d'après J.-P. Demailly, S. Diverio, J. Merker, E. Rousseau, Y.-T. Siu).* Astérisque No. 361 (2014), Exp. No. 1061, vii, 77–113.

[32] J.-P. Rosay, W. Rudin, *Holomorphic maps from* $\mathbb{C}^n$ *to* $\mathbb{C}^n$, Trans. Amer. Math. Soc. 310 (1988), no. 1, 4786

[33] J. Sun *On the Demailly-Semple jet bundles of hypersurfaces in* $\mathbb{P}^3$. Houston J. Math. 41 (2015), no. 1, 73–96.

[34] C. Voisin *On a conjecture of Clemens on rational curves on hypersurfaces.* J. Diff. Geom. 44 (1996) 200213, Correction: J. Diff. Geom. 49 (1998), 601611.

[35] C. Voisin *On some problems of Kobayashi and Lang*
`https://webusers.imj-prg.fr/~claire.voisin/Articlesweb/harvard.pdf`