Minisymposium 13 - Approximationsmethoden für Probleme auf der Sphäre

Fast evaluation of quadrature formulae on the sphere

JENS KEINER (UNIVERSITY OF LÜBECK, INSTITUTE OF MATHEMATICS)

Recently, a fast approximate algorithm for the evaluation of expansions in terms of standard $\mathrm{L}^2\left(\mathbb{S}^2\right)$-orthonormal spherical harmonics at arbitrary nodes on the sphere $\mathbb{S}^2$ has been proposed in [2]. Our aim is to develop a fast algorithm for the adjoint problem, hence the computation of expansion coefficients from sampled data by means of quadrature rules.
We give a formulation in matrix-vector notation and an explicit factorisation of the corresponding spherical Fourier matrix that is based on the first algorithm. Starting from this factorisation, we obtain the corresponding adjoint factorisation and are able to implement the corresponding transform. This 'adjoint' algorithm can be employed to evaluate quadrature rules for arbitrary quadrature nodes and weights on the sphere $\mathbb{S}^2$.
We provide results of test computations with respect to stability, accuracy and performance of the obtained algorithm. As examples, we consider a variety of proposed test functions using classical Gauß-Legendre and Clenshaw-Curtis quadrature rules. Furthermore, we also consider an equidistribution from [3] and the HEALPix pixelation scheme ([1]), each with equal weights for all nodes to obtain a convenient quadrature rule. Especially the HEALPix scheme has great relevance as data storage standard in certain applications like cosmic microwave background estimation.

REFERENCES

[1] K. M. Górski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, M. Reinecke, and M. Bartelmann. Healpix: A framework for high-resolution discretization and fast analysis of data distributed on the sphere. *The Astrophysical Journal*, 622:759–771, 2005.

[2] S. Kunis and D. Potts. Fast spherical Fourier algorithms. *J. Comput. Appl. Math.*, 161:75 – 98, 2003.

[3] W. Freeden, T. Gervens, and M. Schreiner. *Constructive Approximation on the Sphere*. Oxford University Press, Oxford, 1998.